

# Gravitational Collapse in Horndeski Theory

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We study the gravitational collapse of a homogeneous time-dependent scalar field which, besides its coupling to curvature, it is also coupled kinematically to the Einstein tensor. This coupling is part of the Horndeski theory and we investigate the effect of the shift symmetry on the collapsing process. We find that, as the value of the derivative coupling is increasing, the time required for the scalar field to collapse is also increasing, the singularity is protected by a horizon and a black hole is formed while the weak energy condition is satisfied during the collapsing process.

## I. INTRODUCTION

Studying the gravitational collapse of matter to form a neutron star core, Oppenheimer and Volkov [1] and Tolman [2] went beyond Newtonian gravitational theory and suggested that because of high masses and densities general relativistic effects must be considered. They also proposed that, because of the large mass of the neutron star, a non-relativistically degenerate equation of state might be more appropriate and most importantly the gravitational effect of the kinetic energy of the collapsing matter could not be neglected. They based their discussion on a general relativistic treatment of the equilibrium of spherically symmetric distributions of matter. In a subsequent paper Oppenheimer and Snyder [3] studied the non-static solutions resulting from the collapse of a spherically symmetric shell in comoving coordinates.

Since these pioneering works there has been a lot of activity in studying the gravitational collapse of a spherical mass shell and its stability. The strategy was to consider a fluid with some kind of symmetry and then solve the resulting Einstein equations provided that an equation of state is given (for a recent review see [4]). These studies have the disadvantage that they do not capture the dynamics of the collapsing matter. To remedy this, the collapse of matter parameterized by a scalar field has been studied. In this way the dynamics of collapsing matter has been taken under consideration through the scalar field equation. The gravitational collapse of scalar fields in classical General Relativity has been widely studied in literature. Since the early 90's, models of scalar field collapse exhibiting a naked singularity were found numerically by Choptuik [5] and analytically by Christodoulou [6]. These models were violating the so-called Penrose's Cosmic Censorship Conjecture [7]. In these works the scalar field is massless and free of any self-interactions. A class of potentials were found in [8], where smooth initial data evolve to give rise to a naked singularity, although energy conditions may be violated. The role of naked singularities in gravitational lensing was studied in [9].

Collapsing models with homogeneous scalar fields have been discussed, exhibiting a class of potentials determining singularity formation. A crucial role in determining the causal structure is played by the existence, or not, of an apparent horizon during the evolution. Since the singularity is synchronous, the behaviour is quite different from what happens in many examples of matter models exhibiting a central naked singularity. Here, instead, it may be that the singularity located at the boundary of the "ball" of scalar field can be naked. Therefore, the scalar field solution must be matched with a suitable external solution and the behaviour of radial geodesic in the external solution must be studied accordingly.

Conditions under which gravity coupled to self-interacting scalar field yields a singularity formation were found and discussed in [10]. It has been shown that the formation of singularities of the gravitational collapse of homogeneous scalar fields with a potential is completely determined by a condition of integrability of a function related to the energy density of the model. If this function is bounded, apparent horizon formation is impossible during the evolution and a naked singularity is formed. A class of collapsing scalar field models with a non-zero potential was constructed in [11], which resulted in a naked singularity as the collapse end state. The weak energy condition was satisfied by the

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collapsing configuration. It was shown that the formation of either a black hole or a naked singularity as the final state of the dynamical evolution is governed by the collapse rate. It has been seen that the cosmic censorship was violated in dynamical scalar field collapse.

The effects of a non-vanishing cosmological term on the final fate of a spherical homogeneous or inhomogeneous collapsing dust cloud were discussed in [12–14]. It was shown that, depending on the nature of the initial data from which the collapse evolves, the result of collapse is the formation of either a black hole or a naked singularity as the end state of collapse [12]. In [13] it was shown that the cosmological constant term slows down the collapse of matter, limiting also the size of the black hole. The formation of black holes or naked singularities is studied in [15] in a model in which a homogeneous time-dependent scalar field with an exponential potential couples to a four-dimensional gravity with a negative cosmological constant. In [16] the collapse of a charged scalar field in a de Sitter space was studied.

Recently there is an increased interest on the collapsing configuration of a massless minimally coupled scalar field in the presence of a negative cosmological constant. The motivation for such a study were the recent theoretical and experimental developments indicating that the quark-gluon plasma produced at Relativistic Hadron Ion Collider is a strongly interacting liquid and that the produced plasma locally isotropizes over a time scale of  $\tau_{iso} < \frac{1fm}{c}$ . The dynamics of such rapid isotropization in a far-from-equilibrium non-Abelian plasma cannot be described with the standard methods of field theory or hydrodynamics. It has been proposed then, in [17, 18], that such a rapid thermalization can be studied via its gravity dual, the gravitational collapse.

It is important to find the right probe to describe such a process. On the boundary the question is how the field theory reacts to a rapid injection of energy. Then the goal is to find the right gravity methods to describe the time in field theory between the injection of energy at  $t = 0$  and the formation of the quark-gluon plasma, i.e the thermalization time. One such gravity process is the gravitational collapse of a scalar field and a formation of a black hole. Then, the thermalization time is identified with the time needed for the formation of an apparent horizon.

The collapsing rate should then be considered. As the collapse of a homogeneous scalar field progresses, the concentration of high masses and densities in small distances dictates the necessity to consider curvature effects and study their influence on the rate of the collapsing process. To capture these effects the coupling of the collapsing scalar field to curvature must be considered. In this work we will consider a collapsing model with a scalar field which, apart from the usual kinetic coupling to gravity, it is also kinematically coupled to the Einstein tensor. This term belongs to a general class of scalar-tensor gravity theories resulting from the Horndeski Lagrangian [19]. These theories, which were recently rediscovered [20] and written in a simpler form <sup>1</sup>, give second-order field equations and contain as a subest a theory which preserves classical Galilean symmetry [25–27]. The action of the theory is

$$S = \int d^4x \sqrt{-g} (\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5) , \quad (1.1)$$

where

$$\mathcal{L}_2 = G_2(\phi, X) , \quad (1.2)$$

$$\mathcal{L}_3 = -G_3(\phi, X) \square \phi , \quad (1.3)$$

$$\mathcal{L}_4 = G_4(\phi, X) R + G_{4X}(\phi, X) [(\square \phi)^2 - \nabla_\mu \nabla_\nu \phi \nabla^\mu \nabla^\nu \phi] , \quad (1.4)$$

$$\mathcal{L}_5 = G_5(\phi, X) G_{\mu\nu} \nabla^\mu \nabla^\nu \phi - \frac{1}{6} G_{5X}(\phi, X) [(\square \phi)^3 + \nabla_\mu \nabla_\nu \phi \nabla^\nu \nabla^\alpha \phi \nabla^\mu \nabla_\alpha \phi - 3 \square \phi \nabla_\mu \nabla_\nu \phi \nabla^\mu \nabla^\nu \phi] . \quad (1.5)$$

The functions  $G_i$ , with  $i = 2, 3, 4, 5$ , depend on the scalar field  $\phi$  and its kinetic energy  $X = -\frac{1}{2} \nabla^\alpha \phi \nabla_\alpha \phi$  and  $G_{iX}$  denotes just the partial derivative of  $G_i$  with respect to  $X$ ,  $G_{iX} = \frac{\partial G_i}{\partial X}$ .

One of the most important terms in the above action, which is directly connected to our considerations, is the derivative coupling of the scalar field to the Einstein tensor appearing in the Horndeski Lagrangian

$$I = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} - (g^{\mu\nu} - G(\phi) G^{\mu\nu}) \nabla_\mu \phi \nabla_\nu \phi \right] , \quad (1.6)$$

which also includes the canonical kinetic term. Here we have excluded the dependence of the scalar field function  $G(\phi)$  on  $X$ . The above action gives second order field equations due essentially to the vanishing of the divergence of the Einstein tensor. Local solutions for the above action including the coupling function  $G$  constant have been

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<sup>1</sup> See also extensions of Horndeski theory in [21–24].

discussed in several recent papers [28]. Spherically symmetric black hole solutions which are asymptotically anti-de Sitter were found. In these solutions the scalar field usually diverges on the horizon. To circumvent the problem of regularity of local solutions one can break the shift symmetry of the scalar field by introducing a mass term for the scalar field [29, 30]; in this case spherically symmetric black hole solutions have been found. Another way to remedy this problem, while keeping the shift symmetry, is to introduce an additional, mild, linear dependence in the time coordinate for the scalar field [31, 32], and stability issues of this solution were studied in [33]. See also extensions to higher dimensions [34].

The existence of these static local solutions of Horndeski theories motivate us to examine the dynamical formation of these black holes via the gravitational collapse of a scalar field coupled to Einstein tensor. In this work we will study such a collapsing process resulting from an action of the form (1.6). We note that Neutron star configurations resulted also from the action (1.6) were studied in [35].

The work is organized as follows. In Section II we present the general formulation of the theory giving the field equations resulted from the considered action. In Section III we discuss the gravitational collapsing process of reaching the singularity, the formation of apparent horizon and the matching of internal metric with an external Schwarzschild-AdS metric resulting to a formation of an event horizon. This analysis was carried out with zero potential while in Section IV we present the collapse of the scalar field in the presence of a self-interacting potential which breaks the shift symmetry. In Section V we present our conclusions, while in the Appendix we give an analytical solution of the field equations.

## II. GENERAL FORMALISM

We consider the action (1.6) including a cosmological constant and a potential for the scalar field. We also assume that the coupling function  $G(\phi)$  is a constant, independent from the scalar field. Then the action becomes

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R - 2\Lambda}{16\pi G} - \left[ \frac{1}{2}g^{\mu\nu} + \frac{1}{2}\lambda G^{\mu\nu} \right] \partial_\mu \phi \partial_\nu \phi - V(\phi) \right\} . \quad (2.1)$$

Variation of the action above gives the Einstein equations

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G [T_{\mu\nu} + \lambda \Theta_{\mu\nu}] , \quad (2.2)$$

where

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R , \quad (2.3)$$

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - \frac{1}{2}g_{\mu\nu}g^{ab}\partial_a \phi \partial_b \phi - g_{\mu\nu}V(\phi) , \quad (2.4)$$

$$\begin{aligned} \Theta_{\mu\nu} = & -\frac{1}{2}\partial_\mu \phi \partial_\nu \phi R + 2\partial_a \phi \partial_{(\mu} \phi R_{\nu)}^a - \frac{1}{2}G_{\mu\nu}(\partial\phi)^2 + \nabla^a \phi \nabla^b \phi R_{\mu a \nu b} + \nabla_\mu \nabla^a \phi \nabla_\nu \nabla_a \phi \\ & - \nabla_\mu \nabla_\nu \phi \square \phi + g_{\mu\nu} \left[ -\frac{1}{2}\nabla^a \nabla^b \phi \nabla_a \nabla_b \phi + \frac{1}{2}(\square \phi)^2 - \nabla_a \phi \nabla_b \phi R^{ab} \right] , \end{aligned}$$

while the Klein-Gordon equation reads

$$\square \phi - V_\phi = 0 , \quad (2.5)$$

where  $V_\phi$  denotes the derivative with respect to  $\phi$  and  $\square \phi = (-g)^{-1/2} \partial_\mu [(-g)^{1/2} [g^{\mu\nu} + \lambda G^{\mu\nu}] \partial_\nu \phi]$ .

To study the gravitational collapse of the scalar field we assume that it is only time dependent and that the collapsing region follows a metric in comoving coordinates

$$ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right) . \quad (2.6)$$

Then the Klein-Gordon equation becomes

$$\left( 1 - \frac{3\lambda(\dot{a}^2(t) + k)}{a^2(t)} \right) \ddot{\phi}(t) + \left( 3\frac{\dot{a}(t)}{a(t)} - 3\lambda \left( \frac{\dot{a}^3(t)}{a^3(t)} + 2\frac{\dot{a}(t)\ddot{a}(t)}{a^2(t)} + \frac{\dot{a}(t)k}{a^3(t)} \right) \right) \dot{\phi}(t) = -V_\phi . \quad (2.7)$$

The tt-component of the Einstein equations reads

$$\frac{3(k + \dot{a}^2(t))}{a^2(t)} - \Lambda = \frac{4\pi}{M_{pl}^2} \left\{ \left[ 1 - \lambda \left( 6 \frac{\dot{a}^2(t)}{a^2(t)} + 3 \frac{(k + \dot{a}^2(t))}{a^2(t)} \right) \right] \dot{\phi}^2(t) + 2V(\phi) \right\} . \quad (2.8)$$

Next we define the dimensionless quantities

$$\begin{aligned} \tau &\equiv M_{pl} \cdot t \quad , \quad \bar{\lambda} \equiv \lambda \cdot M_{pl}^2 \quad , \quad \bar{k} \equiv \frac{k}{M_{pl}^2} \quad , \quad \bar{\Lambda} \equiv \frac{\Lambda}{M_{pl}^2} \quad , \\ \psi &\equiv \frac{\phi}{M_{pl}} \quad , \quad \bar{V}(\psi) = \frac{V(\psi)}{M_{pl}^4} \quad , \quad \bar{V}_\psi = \frac{V_\phi(\phi)}{M_{pl}^3} \Big|_{\phi=\psi(\tau)M_{pl}} . \end{aligned} \quad (2.9)$$

Equations (2.7) and (2.8) become, in terms of the dimensionless quantities above:

$$\left( 1 - \frac{3\bar{\lambda}(\dot{a}^2(\tau) + \bar{k})}{a^2(\tau)} \right) \ddot{\psi}(\tau) + \left( 3 \frac{\dot{a}(\tau)}{a(\tau)} - 3\bar{\lambda} \left( \frac{\dot{a}^3(\tau)}{a^3(\tau)} + 2 \frac{\dot{a}(\tau)\ddot{a}(\tau)}{a^2(\tau)} + \frac{\dot{a}(\tau)\bar{k}}{a^3(\tau)} \right) \right) \dot{\psi}(\tau) = -\bar{V}_\psi \quad (2.10)$$

and

$$\frac{3(\bar{k} + \dot{a}^2(\tau))}{a^2(\tau)} - \bar{\Lambda} = 4\pi \left\{ \left[ 1 - \bar{\lambda} \left( 6 \frac{\dot{a}^2(\tau)}{a^2(\tau)} + 3 \frac{(\bar{k} + \dot{a}^2(\tau))}{a^2(\tau)} \right) \right] \dot{\psi}^2(\tau) + 2\bar{V}(\psi) \right\} . \quad (2.11)$$

In the next section we will study in details the two equations (2.10) and (2.11).

### III. GRAVITATIONAL COLLAPSE WITH ZERO POTENTIAL

We assume that the collapsing space is flat,  $\bar{k} = 0$ , and we first consider the case in which the scalar field does not have a self-interaction term,  $V(\phi) = 0$ . Then equations (2.10) and (2.11) become

$$-3a^2(\tau)\dot{a}(\tau)\dot{\psi}(\tau) + 3\bar{\lambda}\dot{a}^3(\tau)\dot{\psi}(\tau) - a^3(\tau)\ddot{\psi}(\tau) + 3\bar{\lambda}a(\tau) \left( 2\dot{a}(\tau)\dot{\psi}(\tau)\ddot{a}(\tau) + \dot{a}^2(\tau)\ddot{\psi}(\tau) \right) = 0 , \quad (3.1)$$

$$a^3(\tau) \left( \bar{\Lambda} + 4\pi\dot{\psi}^2(\tau) \right) - 3a(\tau)\dot{a}^2(\tau) \left( 1 + 12\pi\bar{\lambda}\dot{\psi}^2(\tau) \right) = 0 . \quad (3.2)$$

Note that because the action (2.1) is invariant under a shift symmetry (when there is no potential), only derivatives of  $\psi$  appear in the field equations. We solve equation (3.2) with respect to  $\dot{\psi}(\tau)$  and substitute it into equation (3.1). The resulting equation is a function of only  $a(\tau)$  and its derivatives and we solve this equation numerically. We depict the behaviour of the scale factor  $a(\tau)$  in Fig. 1. In the Appendix we solve equations (3.1) and (3.2) analytically and compare the analytical function of the scale factor  $a(\tau)$  with its numerical solution in Fig. 12. The singularity is reached at the singularity time  $\tau_s$  where  $a(\tau_s) = 0$ , and the collapse starts when  $\dot{a}(\tau) < 0$ . The analytic solution in the Appendix proves beyond doubt that the numerical results, in particular the divergence of  $\dot{a}(\tau)$  at  $\tau_s$  are not just numerical artifacts. To see the behaviour of the scalar field  $\psi$  we substitute the numerical solution for the scale factor  $a(\tau)$  to (3.1). In Fig. 2 we can see that it blows up at the singularity, as expected.

In our numerical solutions we have chosen the parameters so that the weak energy condition  $\rho(\tau) + 3p(\tau) \geq 0$ , is satisfied, where

$$\rho(\tau) = T_{00} + \bar{\lambda}\Theta_{00} = \frac{\dot{\psi}^2(\tau)}{2} - \frac{9}{2}\bar{\lambda}\frac{\dot{a}^2(\tau)}{a^2(\tau)} , \quad (3.3)$$

$$p(\tau) = T_{ii} + \bar{\lambda}\Theta_{ii} = \frac{\dot{\psi}(\tau)}{4a^2(\tau)} \left( a^2(\tau)\dot{\psi}^2(\tau) + \bar{\lambda}\dot{a}^2(\tau)\dot{\psi}(\tau) + 2a(\tau)\bar{\lambda}(\dot{\psi}(\tau)\ddot{a}(\tau) + 2\dot{a}(\tau)\ddot{\psi}(\tau)) \right) . \quad (3.4)$$

We observe that, as the absolute value of the derivative coupling  $\bar{\lambda}$  increases, the collapse time as well as the singularity formation time are also increasing. This behaviour of the collapsing scale factor can be understood on the basis of the fact that, in a time dependent background, the derivative coupling for negative values of  $\bar{\lambda}$  acts as a friction term [36–40], so it takes more time to reach the singularity. This effect can be seen more clearly if we switch off the cosmological constant. The dynamics of the collapsing matter is governed by the usual kinetic term of the scalar field and the coupling term of the scalar field to the Einstein tensor. We find that for any value of the derivative coupling the singularity is never reached.

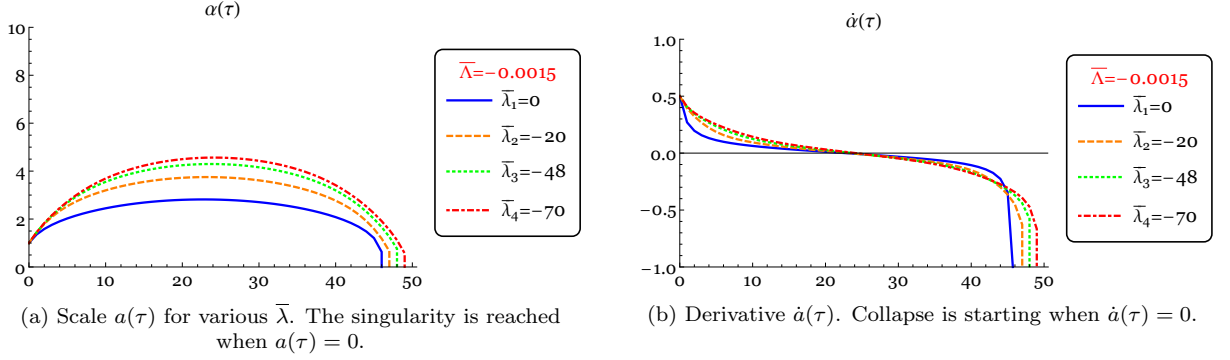
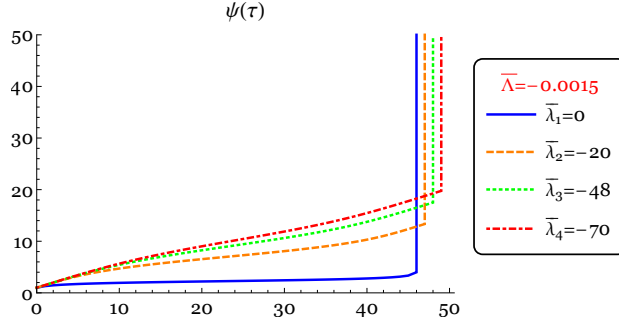


FIG. 1: The behaviour of the scale factor.

FIG. 2: Behaviour of the scalar field  $\psi(\tau)$ 

### A. Formation of apparent horizon

To study the formation of a naked singularity or a black hole we need to investigate the dynamics of apparent horizons. We will analyze under which conditions trapped surfaces form and investigate their properties in connection with the derivative coupling  $\bar{\lambda}$ . Following [11, 15, 41] a definition for the boundary of a possible apparent horizon is obtained from  $g^{\mu\nu} \partial_\mu R \partial_\nu R = 0$  (with  $R(r, t) \equiv ra(t)$ ), which, in the case of the metric (2.6) (with  $k = 0$ ), gives

$$r_{AH}(t) \equiv -\frac{1}{\dot{a}(t)} \Rightarrow r_{AH}(\tau) = -\frac{1}{M_{pl}} \frac{1}{\dot{a}(\tau)} \quad \text{or} \quad \bar{r}_{AH}(\tau) = \frac{1}{\dot{a}(\tau)}. \quad (3.5)$$

To have the formation of an apparent horizon the condition  $r_{AH}(\tau_s) = 0$  or equivalently  $\dot{a}(\tau_s) \rightarrow -\infty$  must be satisfied.

We study first the dynamics of the collapse governed only by the kinetic energy of the scalar field minimally coupled to gravity with a cosmological constant, but where the derivative coupling is absent ( $\bar{\lambda} = 0$ ). In this case the collapsing scale factor  $a(\tau)$  is shown in Fig. 3 and the formation of the apparent horizon in Fig. 4. Comparison of figures 3 and 4 shows that the apparent horizon is formed at the time of the singularity formation and therefore the singularity is protected. This is consistent with the findings in [15] in their special case, where there is no potential for the scalar field. The above results are due to the fact that at  $\tau = \tau_s$  the  $\dot{a}(\tau)$  function is rapidly decreasing towards  $-\infty$ . When the coupling  $\bar{\lambda}$  of the scalar field to the Einstein tensor is switched on, the behaviour of  $\dot{a}(\tau)$  near  $\tau = \tau_s$  is less steep, so one has to check if it actually goes to  $-\infty$  at  $\tau = \tau_s$  and whether this property depends on  $\bar{\lambda}$ . In the Appendix we solve the equations analytically with the results:

$$a(\tau) \approx C_2 \left( \frac{(a_1 + a_2)\pi}{2} - C_1 + \tau \right)^{\frac{2}{3\sqrt{3}}} \quad , \quad \dot{a}(\tau) \approx \frac{2C_2}{3\sqrt{3}} \left( \frac{(a_1 + a_2)\pi}{2} - C_1 + \tau \right)^{\frac{2}{3\sqrt{3}} - 1} \quad (3.6)$$

with  $a_1 = \left( \frac{2}{\sqrt{3}} - 1 \right) \sqrt{|\bar{\lambda}|}$ ,  $a_2 = +\frac{1}{\sqrt{3|\bar{\lambda}|}}$ . We see that we have vanishing scale factor for a suitable value of  $\tau = \tau_s$  and its derivative diverges, since the exponent  $\frac{2}{3\sqrt{3}} - 1$  is negative, signaling the formation of an apparent horizon ( $r_{AH}(\tau = \tau_s) = 0$ ). The above results are  $\bar{\lambda}$  independent, so it turns out that the apparent horizon is formed for any

value of the derivative coupling. Another interesting effect can be seen in Fig. 3. We observe that, as the absolute value of the cosmological constant increases, the collapse and singularity formation time decrease. This is an opposite effect of that we had observed in the case of the increase of the value of the derivative coupling [39]. The cosmological constant  $\Lambda = -3/l^2$  defines a scale  $l$  in the  $AdS_4$  space, therefore the decrease of the collapse and singularity time may be understood as the result of the scalar field having less distance to travel through the  $AdS_4$  space or additionally larger pressure to drive the collapse.

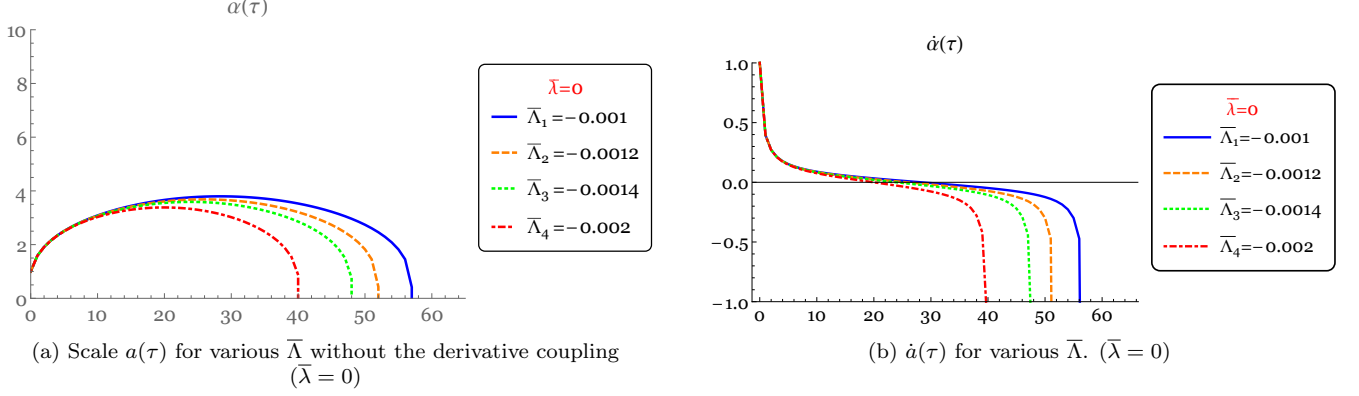


FIG. 3: Formation of the singularity in the presence of a cosmological constant, with no derivative coupling ( $\bar{\lambda} = 0$ ).

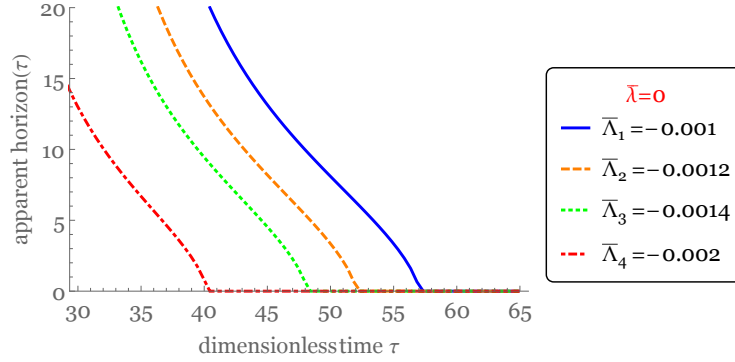


FIG. 4: Apparent horizon  $r_{AH}$  for various  $\bar{\Lambda}$  with no derivative coupling ( $\bar{\lambda} = 0$ ).

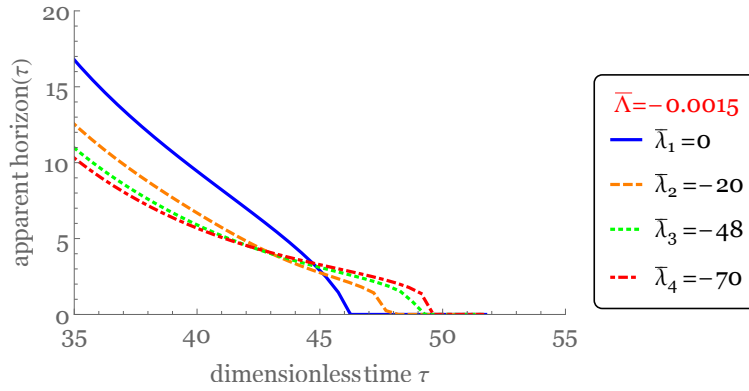


FIG. 5: Formation of the apparent horizon  $r_{AH}(\tau)$

Summarizing, we found that the apparent horizon is formed for any value of the derivative coupling  $\bar{\lambda}$  protecting the singularity and it has similar behaviour to the cosmological constant, indicating that the derivative coupling  $\bar{\lambda}$  acts

as an effective cosmological constant [36–40]. This means that it strengthens the effect of the cosmological constant, however there is also the important difference that the collapse is quite possible for  $\bar{\Lambda} = 0$ , but it is impossible for  $\bar{\Lambda} = 0$ . In the Appendix we offer some explanation of this fact.

### B. Formation of boundary surface

To construct the appropriate event horizon that signals the formation of a black hole we match the metric of our collapsing scalar field with that of a Schwarzschild- $AdS_4$ . The Schwarzschild- $AdS_4$  metric reads

$$ds^2 = - \left( 1 - \frac{2GM_{bh}}{c^2 Y} + \frac{Y^2}{l^2} \right) (cdT)^2 + \left( 1 - \frac{2GM_{bh}}{c^2 Y} + \frac{Y^2}{l^2} \right)^{-1} dY^2 + Y^2(d\theta^2 + \sin^2 \theta d\phi^2) .$$

This metric must be matched with the time dependent metric of the collapsing scalar matter ( $k = 0$ )

$$ds^2 = -(cdt)^2 + a^2(t)dr^2 + a^2(t)r^2(d\theta^2 + \sin^2 \theta d\phi^2) .$$

From the angular part we find  $Y^2 = a^2(t)r^2$  while from the time dependent part we get

$$\left( 1 - \frac{2GM_{bh}}{c^2 Y} + \frac{Y^2}{l^2} \right) = 1 \quad \Rightarrow \quad \left( 1 - \frac{2GM_{bh}}{c^2 a(t)r} + \frac{a^2(t)r^2}{l^2} \right) = 1 \quad \Rightarrow \quad -\frac{2GM_{bh}}{c^2 a(t)r} + \frac{a^2(t)r^2}{\left(-\frac{3}{\bar{\Lambda}}\right)} = 0 .$$

From the above matching conditions we find the time dependent radius

$$r(t) = \frac{1}{a(t)} \frac{(6 \frac{G}{c^2} M_{bh})^{1/3}}{(-\Lambda)^{1/3}} , \quad \text{or} \quad \bar{r}(\tau) = \frac{\bar{\gamma}^{1/3}}{a(\tau)} \quad (3.7)$$

where  $\bar{\gamma} = \frac{6 \frac{G}{c^2} M_{bh}}{-\bar{\Lambda} M_{pl}} \frac{[meters]}{[meters]} = \frac{6 \bar{M}_{bh}}{-\bar{\Lambda} \bar{M}_{pl}}$  is a dimensionless parameter depending on the numerical value  $\bar{M}_{bh}$  of the

black hole mass for fixed  $\bar{\Lambda}$ . The dependence on  $\bar{\Lambda}$  is hiding in the scale factor  $a(\tau)$  of  $\bar{r}(\tau)$ . For the following figures we choose  $M_{bh} = 2 \cdot 10^{-7} kg \simeq 10 M_{pl}$ . We can then draw the above radius in the same plot as the apparent horizon. Their intersection point gives the time of the formation of the boundary surface and consequently the  $r_{boundary}$ . In Fig. 6 we present such an intersection point for  $\bar{\Lambda} = -6$ .

In Fig. 7a we show how the  $r_{boundary}$  changes versus  $\bar{\Lambda}$ , while in Fig. 7b we show the  $\bar{\Lambda}$  dependence of  $\tau_{event}$ . We observe that large absolute values of  $\bar{\Lambda}$  yield black holes with smaller radii (in comparison with the  $\bar{\Lambda} = 0$  case), which, in addition, take more time to form. A combination of these two figures is given in the three-dimensional Fig. 8.

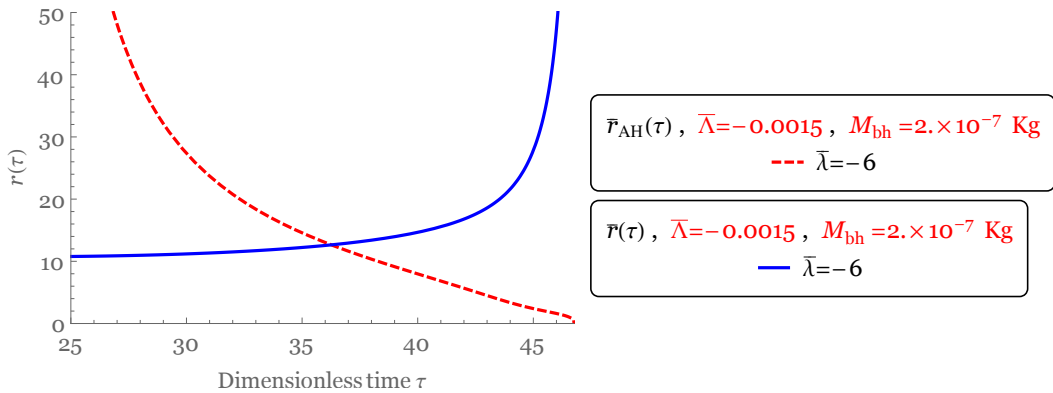


FIG. 6: Formation of  $r_{boundary}$  :  $\bar{r}_{AH}$  and  $\bar{r}(\tau)$  versus time  $\tau$ .

Finally we note that the time of the boundary surface formation remains always smaller than the time of the singularity formation. Comparison of  $\tau_{event}$  and  $\tau_{singularity}$  is given in Fig. 9.

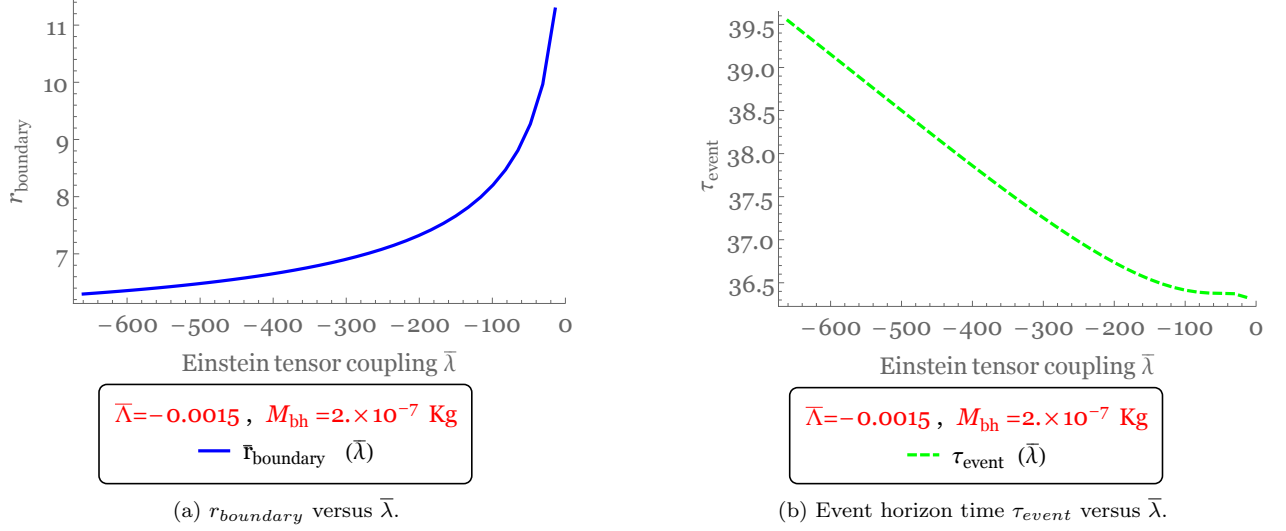


FIG. 7: Event horizon time  $\tau_{\text{event}}$  for various values of the derivative coupling  $\bar{\lambda}$ .

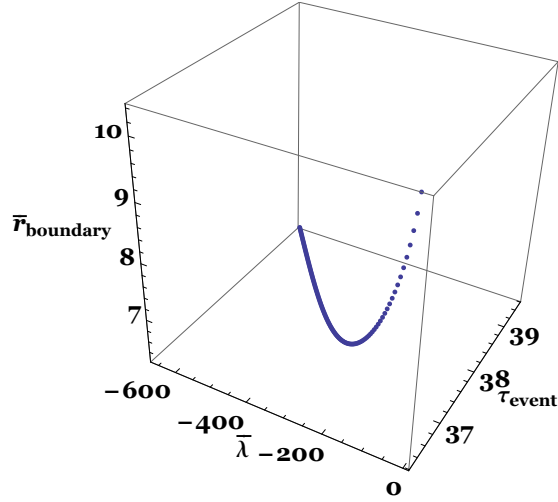


FIG. 8: Depiction of the dependence of  $\tau_{\text{event}}$  and  $r_{\text{boundary}}$  on  $\bar{\lambda}$ .

#### IV. GRAVITATIONAL COLLAPSE WITH NON-ZERO POTENTIAL

In this section we will consider a non-trivial potential which is a function of the scalar field  $\psi(\tau)$ . In this potential  $\psi(\tau)$  appears explicitly in the field equations, so the shift symmetry is no longer preserved. We set  $\bar{k} = 0$ , but we allow for the presence of the derivative coupling. We want to transform the term  $\bar{V}_\psi$  appearing in equation (2.10) to the expression  $\bar{V}_\tau$  through the relation  $\bar{V}_\tau = \bar{V}_\psi \dot{\psi}(\tau)$ . Thus we multiply (2.10) with  $\dot{\psi}(\tau)$  and the result reads

$$-3a^2(\tau)\dot{a}(\tau)\dot{\psi}^2(\tau) + 3\bar{\lambda}\dot{a}^3(\tau)\dot{\psi}^2(\tau) + 3\bar{\lambda}\dot{\psi}(\tau) \left( 2\dot{a}(\tau)\dot{\psi}(\tau)\ddot{a}(\tau) + \dot{a}^2(\tau)\ddot{\psi}(\tau) \right) - a^3(\tau) \left( \bar{V}_\tau + \dot{\psi}(\tau)\ddot{\psi}(\tau) \right) = 0. \quad (4.1)$$

Differentiating equation (2.11) with respect to  $\tau$  we get

$$4\pi a^3(\tau)\bar{V}_\tau + \dot{a}^3(\tau)(3 + 36\pi\bar{\lambda}\dot{\psi}^2(\tau)) - 3a(\tau)\dot{a}(\tau)(1 + 12\pi\bar{\lambda}\dot{\psi}^2(\tau))\ddot{a}(\tau) + 4\pi a^3(\tau)\dot{\psi}(\tau)\ddot{\psi}(\tau) - 36\pi a(\tau)\bar{\lambda}\dot{a}^2(\tau)\dot{\psi}(\tau)\ddot{\psi}(\tau) = 0. \quad (4.2)$$



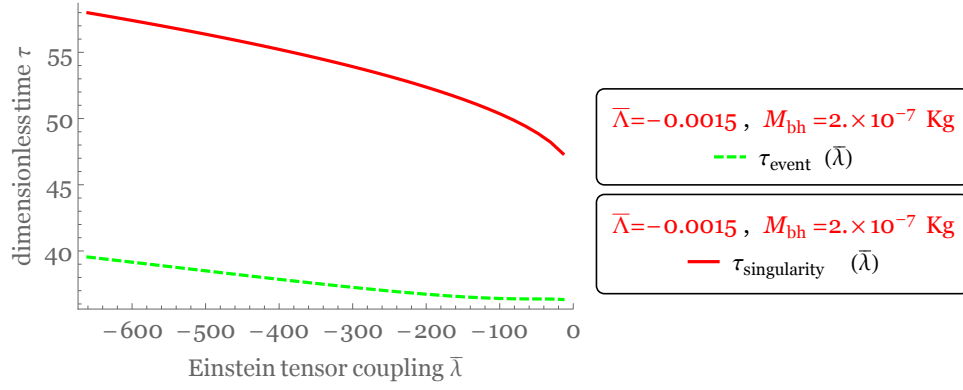


FIG. 9:  $\tau_{event}$  and  $\tau_{singularity}$  versus  $\bar{\lambda}$ .

Next we eliminate  $\bar{V}_\tau$  between (4.2) and (4.1) and get an equation depending on  $\psi(\tau)$ ,  $a(\tau)$  and their derivatives. To solve this equation we need some ansatz for the scale factor  $a$  and we choose the following

$$a(\tau) = \left( \frac{1}{\tau + \bar{\lambda}} + 1 \right)^{1/2}. \quad (4.3)$$

Then, there exists a singularity time  $\tau_s = -\bar{\lambda} - 1$ , where  $a(\tau_s) = 0$ ,  $\dot{a}(\tau_s) \rightarrow -\infty$ . At that time  $r_{AH}(\tau_s) = 0$ , therefore an apparent horizon appears, the singularity is covered by the horizon and a black hole is formed. Given this ansatz for  $a(\tau)$  we may obtain a numerical solution for  $\psi(\tau)$ . Solving the new equation of motion numerically, for  $\bar{\lambda} = -17 \Rightarrow \tau_s = 16$ , we find a solution for  $\psi(\tau)$  which we substitute back to (2.11). We thus get a numerical solution for the potential depending on time, which is depicted in the left hand panel of Fig. 10. With the help of the numerical solution for  $\psi$  versus  $\tau$  we may reconstruct the potential as a function of  $\psi$ . This is depicted in the right hand panel of Fig. 10.

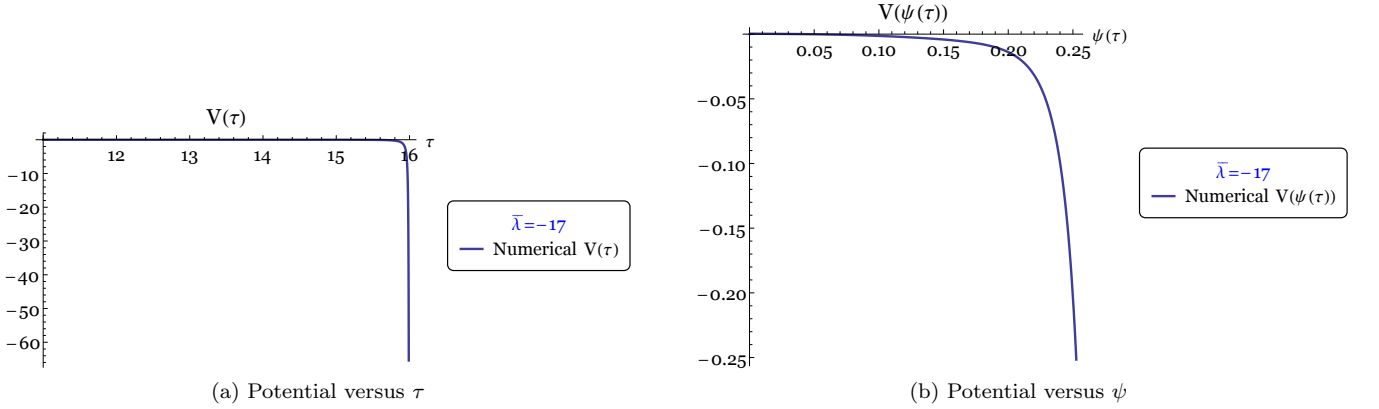


FIG. 10: Form of the potential versus  $\tau$ , as well as versus  $\psi$ .

## V. CONCLUSIONS

We studied the gravitational collapse of a time-dependent scalar field in a theory with negative cosmological constant in which besides its usual coupling to gravity, it is also kinematically coupled to the Einstein tensor. This term is part of the Horndeski theory which gives only second order differential equations and it is characterized by a shift symmetry. We found that, as the absolute value of the derivative coupling  $\bar{\lambda}$  is increased, the time that is needed for the scale factor to reach the singularity also increases. The scalar field blows up at the singularity. We have found an

analytic solution of the field equations showing that an apparent horizon is formed which covers the singularity. We checked that, during the collapsing process, the weak energy condition is satisfied.

Matching the metric of the collapsing scalar field with that of an external Schwarzschild- $AdS_4$  metric we found that an event horizon is formed which bounds the trapped apparent horizon. Varying the derivative coupling  $\bar{\lambda}$  from zero to large absolute values, we found that the radii of the black hole is getting smaller and it takes more time to form. Introducing a self-interaction potential for the scalar field, the shift symmetry is broken, so it is not possible to find an analytic solution of the collapsing process. Choosing a specific ansatz for the scale factor we found numerically that an apparent horizon appears that covers the singularity and a black hole is formed.

It is interesting to extend the above study in the case that the collapsing metric and the scalar field is also space dependent. In this case the field equations cannot be solved analytically and numerical investigation is needed on the line of the work presented in [17, 18]. This will open up the possibility to also include in the study other terms which appear in the Horndeski action and it will provide vital information on the role of Galilean symmetry in the collapsing process.

### Acknowledgments

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### Appendix A: Analytical solution of the field equations

In this appendix we will present an exact solution of the Klein-Gordon and the time Einstein equation. We begin with the Klein-Gordon equation (3.1), multiplied by  $\psi(\tau)$ , where we substitute  $\chi(\tau)$  for the expression  $\psi^2(\tau)$

$$[3\bar{\lambda}a(\tau)\dot{a}^2(\tau) - a^3(\tau)]\dot{\chi}(\tau) + [6\bar{\lambda}\dot{a}^3 + 12\bar{\lambda}a(\tau)\dot{a}(\tau)\ddot{a}(\tau) - 6a^2(\tau)\dot{a}(\tau)]\chi(\tau) = 0, \quad (A1)$$

where a dot denotes a derivative with respect to time. Next we consider the Einstein equation (3.2) performing the same substitutions

$$a^2(\tau)[\bar{\Lambda} + 4\pi\chi(\tau)] - 3\dot{a}^2(\tau)[1 + 12\pi\bar{\lambda}\chi(\tau)] = 0. \quad (A2)$$

Solving equation (A2) for  $\chi(\tau)$  and substituting the result into equation (A1) we get

$$3\dot{a}(\tau)[\bar{\Lambda}a^6(\tau) - (2 + 13\bar{\lambda}\bar{\Lambda})a^4(\tau)\dot{a}^2(\tau) + 9\bar{\lambda}(3 + 2\bar{\lambda}\bar{\Lambda})a^2(\tau)\dot{a}^4(\tau) - 27\bar{\lambda}^2\dot{a}^6(\tau)] \quad (A3)$$

$$- (1 - \bar{\lambda}\bar{\Lambda})a^5(\tau)\ddot{a}(\tau) + 9\bar{\lambda}(1 + \bar{\lambda}\bar{\Lambda})a^3(\tau)\dot{a}^2(\tau)\ddot{a}(\tau) - 54\bar{\lambda}^2a(\tau)\dot{a}^4(\tau)\ddot{a}(\tau)] = 0. \quad (A4)$$

Next we make the substitutions  $\dot{a}(\tau) = H(\tau)a(\tau)$ ,  $\ddot{a}(\tau) = \frac{dH(\tau)}{d\tau}a(\tau) + H(\tau)\dot{a}(\tau) = \left(\frac{dH(\tau)}{d\tau} + H^2(\tau)\right)a(\tau)$ . The result reads

$$\frac{dH(\tau)}{d\tau} = \frac{[\bar{\Lambda} - 3H^2(\tau)][1 - 3\bar{\lambda}H^2(\tau)][1 - 9\bar{\lambda}H^2(\tau)]}{1 - \bar{\lambda}\bar{\Lambda} - 9\bar{\lambda}H^2(\tau) - 9\bar{\lambda}^2\bar{\Lambda}H^2(\tau) + 54\bar{\lambda}^2H^4(\tau)}. \quad (A5)$$

We know from numerical work that the quantities  $\bar{\lambda}$  and  $\bar{\Lambda}$  must be negative, so we substitute  $\bar{\lambda} = -|\bar{\lambda}|$ ,  $\bar{\Lambda} = -|\bar{\Lambda}|$  with the outcome

$$\frac{dH(\tau)}{d\tau} = -\frac{[|\bar{\Lambda}| + 3H^2(\tau)][1 + 3|\bar{\lambda}|H^2(\tau)][1 + 9|\bar{\lambda}|H^2(\tau)]}{1 - |\bar{\lambda}||\bar{\Lambda}|9|\bar{\lambda}|H^2(\tau) + 9|\bar{\lambda}|^2|\bar{\Lambda}|H^2(\tau) + 54|\bar{\lambda}|^2H^4(\tau)}. \quad (A6)$$

This equation is separable and may be solved readily decomposing into partial fractions

$$H[\tau] = \bar{f}(C_1 - \tau), \quad (A7)$$

where  $\bar{f}(t)$  is the inverse function of the expression

$$t = f(H) = \left(\frac{2}{\sqrt{3}} - 1\right)\sqrt{|\bar{\lambda}|}\arctan(3\sqrt{|\bar{\lambda}|}H) + \frac{\arctan(\sqrt{\frac{3}{|\bar{\Lambda}|}}H)}{\sqrt{3|\bar{\Lambda}|}}, \quad (A8)$$

that is  $\bar{f}(f(H)) = H$ . This equation illustrates the fact that the cosmological constant, expressed by  $\bar{\Lambda}$  and the coefficient  $\bar{\lambda}$  are working in the same direction, but one should keep in mind their important difference: one may set  $\bar{\lambda} = 0$ , but one may not set the cosmological constant to zero, since it appears in the denominators.

We may now integrate once more

$$H(\tau) = \frac{\dot{a}(\tau)}{a(\tau)} = \bar{f}(C_1 - \tau) \Rightarrow a(\tau) = C_2 \exp \left[ \int^\tau dx \bar{f}(C_1 - t) \right]. \quad (\text{A9})$$

If we set momentarily  $\bar{\lambda} = 0$ , we get

$$t = f(H) = \frac{\arctan(\sqrt{\frac{3}{|\bar{\Lambda}|}} H)}{\sqrt{3|\bar{\Lambda}|}} \Rightarrow H = \sqrt{\frac{|\bar{\Lambda}|}{3}} \tan(\sqrt{3|\bar{\Lambda}|} t) \Rightarrow \bar{f}(C_1 - \tau) = \sqrt{\frac{|\bar{\Lambda}|}{3}} \tan[\sqrt{3|\bar{\Lambda}|}(C_1 - \tau)]. \quad (\text{A10})$$

Then

$$a(\tau) = C_2 \exp \left[ \int^\tau dt \sqrt{\frac{|\bar{\Lambda}|}{3}} \tan[\sqrt{3|\bar{\Lambda}|} t] \right] = C_2 \exp \left[ \frac{1}{3} \ln \left| \cos[\sqrt{3|\bar{\Lambda}|}(C_1 - \tau)] \right| \right] = C_2 \left| \cos[\sqrt{3|\bar{\Lambda}|}(C_1 - \tau)] \right|^{1/3}. \quad (\text{A11})$$

It is evident that

$$a \left( \tau = C_1 + \frac{\pi}{2\sqrt{3|\bar{\Lambda}|}} \right) = 0,$$

as well as that

$$\dot{a} \left( \tau = C_1 + \frac{\pi}{2\sqrt{3|\bar{\Lambda}|}} \right) = \frac{C_2 \sqrt{3|\bar{\Lambda}|}}{3} \frac{\sin[\sqrt{3|\bar{\Lambda}|}(C_1 - \tau)]}{\cos^{2/3}[\sqrt{3|\bar{\Lambda}|}(C_1 - \tau)]} \Big|_{\tau=C_1+\frac{\pi}{2\sqrt{3|\bar{\Lambda}|}}} = -\infty.$$

It is instructive to give the plot of the function  $t = f(H)$  for the two cases  $|\bar{\lambda}| = 0.0$  and  $|\bar{\lambda}| = 10.0$ , while  $|\bar{\Lambda}| = 0.001$  throughout. The two curves appear similar. The logarithmic behaviour within equation (A10) is mainly due to the

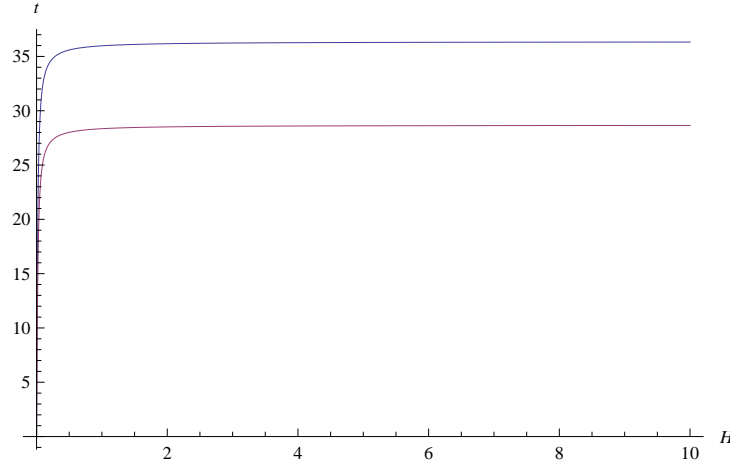


FIG. 11: Function  $t = f(H)$  for  $|\bar{\Lambda}| = 0.001$  and either  $|\bar{\lambda}| = 0.0$  (lower curve) or  $|\bar{\lambda}| = 10.0$  (upper curve).

large  $H$  behavior of  $f(H)$ . In fact the inverse function  $\bar{f}(C_1 - t) = \sqrt{\frac{|\bar{\Lambda}|}{3}} \tan[\sqrt{3|\bar{\Lambda}|}(C_1 - t)]$  may be approximated by  $\frac{1}{\frac{\pi}{2} - \sqrt{3|\bar{\Lambda}|}(C_1 - t)}$ , when the argument  $\sqrt{3|\bar{\Lambda}|}(C_1 - t)$  approaches  $\frac{\pi}{2}$  from below. Thus the relevant integral becomes

$$a(\tau) \approx C_2 \exp \left[ -\frac{1}{3} \int^\tau d[\sqrt{3|\bar{\Lambda}|}(C_1 - t)] \frac{1}{\frac{\pi}{2} - \sqrt{3|\bar{\Lambda}|}(C_1 - t)} \right]$$

$$= C_2 \exp \left[ \frac{1}{3} \ln \left\{ \frac{\pi}{2} - \sqrt{3|\bar{\Lambda}|} (C_1 - \tau) \right\} \right] = C_2 \left[ \frac{\pi}{2} - \sqrt{3|\bar{\Lambda}|} (C_1 - \tau) \right]^{1/3}. \quad (\text{A12})$$

This approximate expression vanishes at the same point as the exact  $a(\tau)$  and has  $\dot{a}(\tau) = -\infty$  at this point.

The problem is how to generalize the above results to the case with non-zero  $|\bar{\lambda}|$ . Consider the equation  $H = \frac{1}{b} \tan \frac{t}{a}$ . The inverse function is  $t = a \arctan(bH)$ . Now, when  $\frac{t}{a}$  approaches  $\frac{\pi}{2}$ , the two equations read

$$H \approx \frac{1}{b \left( \frac{\pi}{2} - \frac{t}{a} \right)} \Leftrightarrow t \approx a \left( \frac{\pi}{2} - \frac{1}{bH} \right).$$

In our case we have

$$t = a_1 \arctan(b_1 H) + a_2 \arctan(b_2 H) \quad (\text{A13})$$

where

$$a_1 = \left( \frac{2}{\sqrt{3}} - 1 \right) \sqrt{|\bar{\lambda}|}, \quad b_1 = 3\sqrt{|\bar{\lambda}|}, \quad a_2 = +\frac{1}{\sqrt{3|\bar{\Lambda}|}}, \quad b_2 = \sqrt{\frac{3}{|\bar{\Lambda}|}}. \quad (\text{A14})$$

The approximate form of the function reads

$$t \approx a_1 \left( \frac{\pi}{2} - \frac{1}{b_1 H} \right) + a_2 \left( \frac{\pi}{2} - \frac{1}{b_2 H} \right) = \frac{(a_1 + a_2)\pi}{2} - \left[ \frac{a_1}{b_1} + \frac{a_2}{b_2} \right] \frac{1}{H} \Rightarrow H = \frac{\frac{a_1}{b_1} + \frac{a_2}{b_2}}{\frac{(a_1 + a_2)\pi}{2} - t}. \quad (\text{A15})$$

Thus we may integrate to find the exponent

$$\int^\tau H|_{C_1-t} = \int^\tau \frac{\frac{a_1}{b_1} + \frac{a_2}{b_2}}{\frac{(a_1 + a_2)\pi}{2} - C_1 + t} \approx \ln \left( \frac{(a_1 + a_2)\pi}{2} - C_1 + \tau \right)^{\left( \frac{a_1}{b_1} + \frac{a_2}{b_2} \right)}, \quad (\text{A16})$$

so the exponent is the expression

$$\gamma \equiv \frac{a_1}{b_1} + \frac{a_2}{b_2}. \quad (\text{A17})$$

That is,

$$a(\tau) \approx C_2 \left( \frac{(a_1 + a_2)\pi}{2} - C_1 + \tau \right)^\gamma \Rightarrow \dot{a}(\tau) \approx C_2 \gamma \left( \frac{(a_1 + a_2)\pi}{2} - C_1 + \tau \right)^{\gamma-1}. \quad (\text{A18})$$

For example, in the  $|\bar{\lambda}| = 0$  case the  $|\bar{\lambda}|$  terms do not contribute, while the relevant constants read  $a_2 = \frac{1}{\sqrt{3|\bar{\Lambda}|}}$ ,  $b_2 = \sqrt{\frac{3}{|\bar{\Lambda}|}}$ , so the exponent equals  $\gamma = \frac{a_2}{b_2} = \frac{\frac{1}{\sqrt{3|\bar{\Lambda}|}}}{\sqrt{\frac{3}{|\bar{\Lambda}|}}} = \frac{1}{3}$  and

$$a(\tau) \approx C_2 \left( \frac{(a_1 + a_2)\pi}{2} - C_1 + \tau \right)^{\frac{1}{3}}, \quad \dot{a}(\tau) \approx \frac{C_2}{3} \left( \frac{(a_1 + a_2)\pi}{2} - C_1 + \tau \right)^{-\frac{2}{3}}. \quad (\text{A19})$$

This means that, for  $\frac{(a_1 + a_2)\pi}{2} - C_1 + \tau = 0$ , the scale factor  $a(\tau)$  vanishes, while its derivative diverges. For  $|\bar{\lambda}| > 0$ :

$$\gamma \equiv \frac{\left( \frac{2}{\sqrt{3}} - 1 \right) \sqrt{|\bar{\lambda}|}}{3\sqrt{|\bar{\lambda}|}} + \frac{1}{3} = \frac{\frac{2}{\sqrt{3}} - 1}{3} + \frac{1}{3} = \frac{2}{3\sqrt{3}}, \quad (\text{A20})$$

and

$$a(\tau) \approx C_2 \left( \frac{(a_1 + a_2)\pi}{2} - C_1 + \tau \right)^{\frac{2}{3\sqrt{3}}} \Rightarrow \dot{a}(\tau) \approx \frac{2C_2}{3\sqrt{3}} \left( \frac{(a_1 + a_2)\pi}{2} - C_1 + \tau \right)^{\frac{2}{3\sqrt{3}}-1}. \quad (\text{A21})$$

Thus the scale factor vanishes for a suitable value of  $\tau = \tau_s$  and its derivative diverges to  $-\infty$ , since the exponent  $\frac{2}{3\sqrt{3}} - 1$  is negative, signaling the formation of an apparent horizon ( $r_{AH}(\tau = \tau_s) = 0$ ). We note that the result is independent of the coupling  $\bar{\lambda}$ .

Another way to look at the problem, without involving the analytic approximations used before, might be to fit the function depicted in figure 11 by the expression  $t_{fit} = a_3 \arctan(b_3 H_{fit})$ , whose inverse function reads:  $H_{fit} = \frac{1}{b_3} \tan \frac{t_{fit}}{a_3}$ , as we have already said. This function can be easily integrated yielding

$$a(\tau) \approx C_2 \left( \frac{\pi a_3}{2} - C_1 + \tau \right)^{\frac{b_3}{a_3}}. \quad (\text{A22})$$

The result of the fit reads:  $a_3 = 0.6639$ ,  $b_3 = 0.2555$ , so  $\frac{b_3}{a_3} \approx 0.3849$ , a value quite close to the numerical value of  $\frac{2}{3\sqrt{3}} \approx 0.3849$  found before. Adopting the values of  $a_3$  and  $b_3$  just found, one may also fit the scale factor via a suitable choice of  $C_1$  and  $C_2$  in equation

$$a(\tau) = C_2 \exp \left[ \frac{1}{b_3} \int dt \tan \left( \frac{1}{a_3} (C_1 - t) \right) \right] \quad (\text{A23})$$

and the result is very close to the numerical solution, as depicted in figure 12.

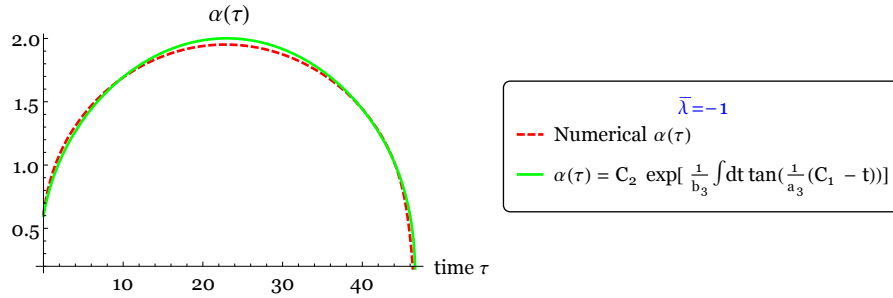


FIG. 12: Numerical solution of the scale  $a(\tau)$  and a fit function for the coupling  $\bar{\lambda} = -1$ .

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